Refined Solutions of Externally Induced Sloshing in Half-Full Spherical Containers

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Abstract: A mathematical model is developed for calculating liquid sloshing effects such as hydrodynamic pressures and forces in half-full spherical containers under arbitrary external excitation. The velocity potential is expressed in a series form, where each term is the product of a time function and the associated spatial function. Because of the spherical configuration, the problem is not separable and the associated spatial functions are nonorthogonal. Application of the boundary conditions results in a system of coupled nonhomogeneous ordinary linear differential equations. The system is solved numerically, implementing a typical fourth-order Runge-Kutta integration scheme. The proposed simple methodology is capable of predicting sloshing effects in half-full spherical containers under arbitrary external excitation in an accurate manner. Hydrodynamic pressures and horizontal forces on the wall of a spherical container are calculated for real earthquake ground motion data. Dissipation effects are included in the present formulation, and their influence on the response is examined. Finally, it is shown that for the particular case of harmonic excitation, the system of ordinary differential equations results in a system of linear algebraic equations, which yields an elegant semianalytical solution.

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Introduction

The sloshing problem has been considered a typical linear eigenvalue problem, which represents the oscillations of the free surface of an ideal liquid inside a container (Lamb 1945). Those free oscillations are described through a velocity potential function satisfying: (1) the Laplace equation within the fluid; (2) the no-flow condition on the tank wall; and (3) the kinematic and dynamic free-surface conditions. Considering small-amplitude free oscillations (i.e., linearized conditions on the free surface), and assuming a harmonic solution, the above equations result in a typical eigenvalue problem. The solution provides the natural frequencies of fluid oscillation (sloshing frequencies) and the corresponding sloshing modes, which strongly depend on the shape of the container. For rectangular and vertical-cylindrical containers the eigenvalue-sloshing problem can be solved analytically, using separation of variables (Currie 1974), and the corresponding sloshing modes are mutually orthogonal and uncoupled. For other geometries (e.g., horizontal cylinders or spheres) exact analytical solutions may not be available, and the use of numerical methods becomes necessary, in addition to experimental measurements.

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Quite often, calculation of sloshing frequencies may not be sufficient in engineering applications and the response of the liquid container under external excitation is necessary. In many practical applications in civil, mechanical, aerospace, and marine engineering, hydrodynamic pressures and the corresponding forces due to sloshing need to be calculated. In particular, earthquake-induced sloshing has been recognized as an important issue toward safeguarding the structural integrity of liquid storage tanks, and has been the subject of numerous analytical, numerical, and experimental works. The pioneering works of Housner (1957, 1963) presented a solution for the hydrodynamic effects in nondeformable vertical cylinders and rectangles. The solution was split in two parts, namely, the "impulsive" part and the "convective" part. This concept constitutes the basis for the API 650 standard provisions (Appendix E) for vertical cylindrical tanks (American Petroleum Institute 1995). Veletsos (1974), Veletsos and Yang (1977), Haroun and Housner (1981), and Haroun (1983), have extended this formulation to include the effects of shell deformation, and its interaction with hydrodynamic effects. More recently, the case of uplifting of unanchored tanks as well as soil-structure interaction effects have been studied extensively, in the papers by Peek (1988), Natsiavas (1988), Veletsos and Tang (1990), and Malhotra (1995). Notable contributions on the seismic response of anchored and unanchored liquid storage tanks have been presented by Fisher (1979), Rammerstorfer et al. (1988), and Fisher et al. (1991), with particular emphasis on design implications. Apparently, those papers constitute the basis for the seismic design provisions concerning vertical cylindrical tanks in Eurocode 8 (CEN 1998) (EC8—Part 4.3—Annex A). In addition to the numerous analytical/numerical works, notable experimental contributions on this subject have been reported (Niwa and Clough 1982; Manos and Clough 1982). The reader is referred to the review paper of Rammerstorfer et al. (1990) for a thorough presentation and a concise literature review of liquid storage tank response under seismic loads, including fluid-structure and soil-structure interaction effects. In a recent publication, Ibrahim et al. (2001) have reviewed a large number of publications on sloshing dynamics, addressing special issues such as nonlinear sloshing, equivalent mechanical models, stochastic excitation, deformable wall effects, hydrodynamic impact, or sloshing in low gravitation fields.

The above studies have been concentrated on vertical-cylindrical tanks, as well as on rectangular tanks. On the other hand, spherical vessels have significant applications (e.g., in chemical plants and refineries), and their sloshing response under strong seismic events is of particular interest for a reliable estimate of the total horizontal force and the corresponding overturning moment. It is interesting to note that the amount of theoretical and numerical works concerning sloshing in spherical containers is quite limited, when compared with the large number of works in vertical cylinders. Furthermore, in current design practice, the recent provisions of Eurocode 8 (Annex A of EC8, Part 4) are particularly detailed concerning sloshing hydrodynamic effects due to earthquake excitation in vertical cylinders and rectangles, whereas the case of spherical vessel is not considered. Similarly, the API provisions for the seismic design of liquid storage tanks (Appendix E of API Standard 650) refer exclusively to vertical cylinders.

Budiansky (1960) has examined sloshing effects in circular canals and spheres, using space transformations to map the initial circular or spherical region to a more convenient plane region. The flow field was described by a set of integral equations, which was solved using a Galerkin-type solution. Numerical values of modal frequencies and hydrodynamic forces were presented for a spherical container. Abramson et al. (1963) presented test data on spherical tank sloshing, which were in good agreement with Budiansky's theory. Moiseev and Petrov (1966) described the application of the Ritz variational method for the numerical calculation of sloshing frequencies in vessels of various geometries, including the case of a spherical container. Fox and Kutler (1981, 1983) obtained upper and lower bounds for the values of sloshing frequencies in a semicircular canal (the two-dimensional analogue of a spherical tank) using conformal mapping and the method of intermediate problems. However, most of the applied conformal mapping techniques require complicated transformations, while the complex variable methods are not applicable in full three-dimensional problems. McIver (1989) considered horizontal cylindrical and spherical containers, filled up to an arbitrary height. Choosing appropriate coordinate systems, so that the container walls and the free surface coincide with coordinate lines or surfaces, McIver reformulated the eigenvalue-sloshing problem in terms of integral equations, which were solved numerically. McIver and McIver (1993) presented simple analytical methods to obtain upper and lower bounds of sloshing frequencies in horizontal cylinders, which were found to be in good agreement with the results from a boundary element numerical solution. More recently, Ortiz and Barhorst (1997, 1998), using a numerical formulation, have addressed large displacement nonlinear sloshing in two-dimensional rigid circular containers interacting with a flexible structure.

Generally, the analysis of sloshing in spherical vessels filled up to an arbitrary height requires a numerical solution. However, for the particular case of a half-full sphere it is possible to develop an analytical solution. Evans and Linton (1993) presented a series-type (semianalytical) solution of the eigenvalue-sloshing problem in hemispherical containers, minimizing the computational effort. Assuming a harmonic solution with respect to time, the velocity potential was expanded in terms of nonorthogonal bounded harmonic spatial functions. Application of the boundary conditions on the tank wall and the free surface resulted in a homogeneous system of algebraic equations, which was solved in terms of the sloshing frequencies.

The present work is aimed at calculating the response in half-full spherical containers under external excitation, with an emphasis on seismic ground motion, extending the analytical formulation proposed by Evans and Linton (1993). In particular, the objective of the paper is the solution of externally induced liquid sloshing in half-full spherical tanks through a semianalytical manner, without implementing finite difference or finite element approximations. The well-known separation-of-variables approach, which works effectively in vertical cylindrical and rectangular configurations, cannot be applied successfully in spherical vessels, since the governing equation with the associated boundary conditions are not separable. The potential solution is divided in two parts as suggested in previous works (Miles 1958; Abramson 1966; Bauer 1984; Isaacson and Subbiach 1991): (1) a ''uniform motion'' part, trivially obtained, representing the liquid motion which follows the external excitation source and (2) a part related to sloshing, representing the relative fluid motion within the container. The velocity potential related to sloshing is expanded in bounded series in terms of arbitrary time functions and their associated nonorthogonal spatial functions. Using the above solution methodology, the problem reduces to a system of ordinary linear differential equations, which is solved numerically. Dissipation effects are considered through the introduction of a damping matrix. Subsequently, the corresponding hydrodynamic pressures and forces are computed in a simple and efficient manner.

The present paper is organized in the following manner. In the section ''Theoretical Formulation and Solution,'' the mathematical formulation and the solution methodology of the half-full sphere response problem under external excitation is described in detail. The important case of harmonic excitation is studied separately in the section ''Solution for Harmonic Excitation'' and explicit expressions for the hydrodynamic pressures and forces on the vessel are derived. Numerical results are presented in the section ''Numerical Results and Discussion.'' The validity and the accuracy of the proposed methodology in terms of the sloshing frequency values are examined first. Subsequently, semianalytical results for harmonic excitation are derived and, finally, numerical results for half-full spherical containers under arbitrary excitation are presented. In the section ''Conclusions,'' a brief summary and some important concluding remarks are stated.

Theoretical Formulation and Solution

In the present analysis of half-full spherical tanks, the flow is considered incompressible and irrotational. The vessel wall is assumed rigid (nondeformable), since in most applications, spherical vessels are rather thick to resist high levels of internal pressure. The total velocity potential $\Phi=\Phi(x,y,z,t)$ satisfies the Laplace equation within the fluid domain, subjected to the free-surface dynamic and kinematic boundary conditions, and the kinematic condition at the container wall. The container undergoes an arbitrary motion in the direction of a specific axis (say the x axis) with displacement $X(t)$, as shown in Fig. 1. The acceleration of the external excitation $\ddot{X}(t)$ and the resulting hydrodynamic force $F(t)$ may be considered as the input and the output of the system, respectively. The amplitude of the external excitation and the resulting free surface elevation (sloshing wave) are assumed to be sufficiently small to allow linearization of the problem. The formulation results in a system of second order ordinary linear differential equations.

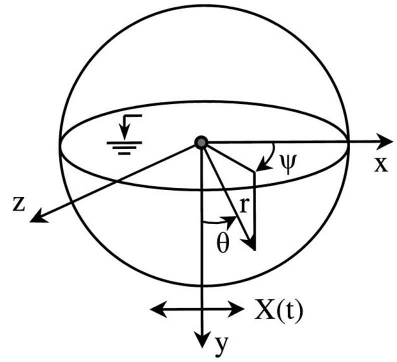

Fig. 1. Configuration of half-full spherical container

Half-Full Spherical Container Under External Excitation

The fluid is contained in a rigid spherical vessel of radius R. The vessel is half full, the origin of the coordinate system xyz, is set at the center of the sphere, which is also the center of the free surface disk and the y-axis points vertically downwards. The complete configuration of the system is shown in Fig. 1, and the geometry is described in terms of the spherical coordinates r, θ, and Ψ, related to Cartesian coordinates x, y, and z as follows:

$$x = r\sin\theta\cos\Psi, \quad y = r\cos\theta, \quad \text{and} \quad z = r\sin\theta\sin\Psi \tag{1}$$

It is assumed that the fluid inside the container is inviscid and the flow is described by a velocity potential function $\Phi(r,\theta,\Psi,t)$, which satisfies Laplace equation

$$\nabla^2\Phi = \frac{\partial}{\partial r}\left(r^2\frac{\partial\Phi}{\partial r}\right) + \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Phi}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2\Phi}{\partial\Psi^2} = 0$$

$$r < R, \quad 0 \leqslant \theta < \pi/2, \quad 0 \leqslant \Psi \leqslant 2\pi \tag{2}$$

The velocity potential is also subjected to the linearized dynamic and kinematic free surface conditions

$$\frac{\partial\Phi}{\partial t} - g\eta = 0, \quad \text{at } \theta = \pi/2, \quad r < R, \quad 0 \leqslant \Psi \leqslant 2\pi \tag{3}$$

and

$$\frac{1}{r}\frac{\partial\Phi}{\partial\theta} + \frac{\partial\eta}{\partial t} = 0, \quad \text{at } \theta = \pi/2, \quad r < R, \quad 0 \leqslant \Psi \leqslant 2\pi \tag{4}$$

respectively, where g=gravitational constant and $\eta = \eta(r,\Psi,t)$=free surface elevation. The combination of Eqs. (3) and (4) leads to the following mixed boundary condition:

$$\frac{\partial^2\Phi}{\partial t^2} + \frac{g}{r}\frac{\partial\Phi}{\partial\theta} = 0, \quad \text{at } \theta = \pi/2, \quad r < R, \quad 0 \leqslant \Psi \leqslant 2\pi \tag{5}$$

Finally, the sloshing potential should satisfy the kinematic condition at the hemispherical wall of the container

$$\frac{\partial\Phi}{\partial r} = \dot{X}(t)\sin\theta\cos\Psi, \quad \text{at } r = R, \quad 0 \leqslant \theta < \pi/2, \quad 0 \leqslant \Psi \leqslant 2\pi \tag{6}$$

Subsequently, the velocity potential $\Phi(r,\theta,\Psi,t)$ is decomposed in two parts

$$\Phi(r,\theta,\Psi,t) = f(r,\theta,\Psi,t) + \varphi(r,\theta,\Psi,t) \tag{7}$$

where $f(r,\theta,\Psi,t)$ and $\varphi(r,\theta,\Psi,t)$=“uniform motion” velocity potential and the potential related to sloshing, respectively. The velocity potential f corresponds to a rigid body motion of the fluid, which follows exactly the motion of the external excitation source, and the velocity potential φ represents the relative motion of the fluid particles within the container due to sloshing.

General Solution for Half-Full Spherical Containers

The externally induced sloshing problem consists of the governing Laplace Eq. (2), and the boundary conditions (5) and (6). Assuming an arbitrary motion of the external source in the x direction (Fig. 1) and decomposition of the total potential in the form of Eq. (7), the uniform motion potential f is taken as

$$f(r,\theta,\Psi,t) = f(x,t) = \dot{X}(t)x = \dot{X}(t)r\sin\theta\cos\Psi \tag{8}$$

which satisfies the Laplace Eq. (2), and the following conditions:

$$\frac{\partial f}{\partial x} = \dot{X}(t), \quad \frac{\partial f}{\partial z} = 0, \quad \text{and} \quad \frac{\partial f}{\partial y} = 0 \tag{9}$$

Thus, the unknown potential φ associated with sloshing, should satisfy the Laplace Eq. (2) within the fluid region and the following boundary conditions:

$$\frac{\partial^2\varphi}{\partial t^2} + \frac{g}{r}\frac{\partial\varphi}{\partial\theta} = -\dddot{X}(t)r\cos\Psi, \quad \text{at } \theta = \pi/2$$

$$r < R, \quad 0 \leqslant \Psi \leqslant 2\pi \tag{10}$$

and

$$\frac{\partial\varphi}{\partial r} = 0, \quad \text{at } r = R, \quad 0 \leqslant \theta \leqslant \pi/2, \quad 0 \leqslant \Psi \leqslant 2\pi \tag{11}$$

A solution for the unknown function φ is considered in a series form as

$$\varphi(r,\theta,\Psi,t) = \sum_{n=m}^{\infty} \dot{q}_n(t)\varphi_n(r,\theta,\Psi;m)$$

$$r < R, \quad 0 \leqslant \theta < \pi/2, \quad 0 \leqslant \Psi \leqslant 2\pi \tag{12}$$

where $q_n(t)$=unknown arbitrary time functions and φ_n=corresponding spatial functions obtained from the general solution of Laplace equation in spherical coordinates, given by

$$\varphi_n(r,\theta,\Psi;m) = P_n^m(\mu)r^n\cos(m\Psi)$$

$$r < R, \quad 0 \leqslant \theta < \pi/2, \quad 0 \leqslant \Psi \leqslant 2\pi \tag{13}$$

with $\mu = \cos\theta$. In the above expression, $P_n^m(\mu)$ (with $m \leqslant n$ and $n = 0,1,2,3...$,), are the associated Legendre functions. Considering the form of the combined free surface condition, expressed in Eq. (10), it can be readily shown that only the terms corresponding to $m = 1$ are nonzero, whereas all other terms vanish. Furthermore, as suggested by Evans and Linton (1993), the expression for the unknown potential is rewritten in the form

$$\varphi(r,\theta,\Psi,t) = \sum_{n=1}^{\infty} [\dot{q}_{2n-1}(t)P_{2n-1}^1(\mu)r^{2n-1} + \dot{q}_{2n}(t)P_{2n}^1(\mu)r^{2n}]\cos\Psi \tag{14}$$

separating odd and even terms of the series. Substituting Eq. (14) into Eq. (10) the following relations are obtained:

$$q_2(t) = \frac{1}{3g}\ddot{q}_1(t) - \frac{1}{3g}\ddot{X}(t) \tag{15}$$

and

$$q_{2n}(t)=\frac{1}{(2n+1)g}\ddot{q}_{2n-1}(t), \quad \text{for } n>1 \tag{16}$$

Eqs. (15) and (16) are substituted back into Eq. (14) and then the boundary condition at the container wall, expressed by Eq. (11), is applied to yield

$$\sum_{n=1}^{\infty}\left\{\frac{2n}{2n+1}\frac{R}{g}P_{2n}^{1}(\mu)\ddot{q}_{2n-1}(t)+(2n-1)\right.$$

$$\left.\times P_{2n-1}^{1}(\mu)q_{2n-1}(t)\right\}R^{2n-2}=\frac{2}{3}\frac{R}{g}P_{2}^{1}(\mu)\ddot{X}(t) \tag{17}$$

Subsequently, applying on Eq. (17) the integral operator

$$I_s=\int_0^1\cdots P_{2s-1}^{00001}(\mu)d\mu, \quad s=1,2,3,... \tag{18}$$

the following infinite system of second-order ordinary linear differential equations is obtained:

$$\sum_{n=1}^{\infty}\left\{\frac{2n}{2n+1}\frac{R^{2n-1}}{g}\int_0^1 P_{2n}^{1}(\mu)P_{2s-1}^{1}(\mu)d\mu\right\}\ddot{q}_{2n-1}(t)$$

$$+\sum_{n=1}^{\infty}\left\{(2n-1)R^{2n-2}\right.$$

$$\left.\times\int_0^1 P_{2n-1}^{1}(\mu)P_{2s-1}^{1}(\mu)d\mu\right\}q_{2n-1}(t)$$

$$=\frac{2}{3}\frac{R}{g}\int_0^1 P_{2}^{1}(\mu)P_{2s-1}^{1}(\mu)d\mu\ddot{X}(t), \quad s=1,2,3,... \tag{19}$$

The above system of equations can be rewritten in the following matrix form:

$$[\mathbf{M}]\{\ddot{q}\}+[\mathbf{K}]\{q\}=\{\gamma\}\ddot{X} \tag{20}$$

where the following identity is employed:

$$\int_0^1 P_{2n}^{1}(\mu)P_{2s}^{1}(\mu)d\mu=\delta_{\text{ns}}\frac{4s^2+2s}{4s+1} \tag{21}$$

$[\mathbf{M}]$=nonsymmetric square matrix with elements

$$M_{\text{sn}}=\frac{2n}{2n+1}\frac{R^{2n-1}}{g}\int_0^1 P_{2n}^{1}(\mu)P_{2s-1}^{1}(\mu)d\mu$$

$$n=1,2,3,..., \quad \text{and } s=1,2,3,... \tag{22}$$

$[\mathbf{K}]$=diagonal matrix with elements

$$K_{\text{nn}}=(2n-1)\frac{4n^2-2n}{4n-1}R^{2n-2} \quad n=1,2,3,... \tag{23}$$

$\{\gamma\}$=vector with elements

$$\gamma_s=\frac{2R}{3g}\int_0^1 P_{2}^{1}(\mu)P_{2s-1}^{1}(\mu)d\mu \quad s=1,2,3,... \tag{24}$$

and $\{q\}$=unknown vector with components $q_{2n-1}(t)$, $n=1,2,\ldots$. The solution of Eq. (20) can be performed through a typical time marching numerical scheme, and leads to the calculation of the arbitrary time functions $q_{2n-1}(t)$ and their first and second derivatives.

The system of equations, Eq. (20), expresses the dynamic equilibrium of the system, where $\{q\}$ is the vector of unknown generalized coordinates, $[\mathbf{M}]$ and $[\mathbf{K}]$ may be considered as the mass and stiffness matrices of the system, respectively, and $\{\gamma\}$ is the vector expressing the contribution (participation) of external excitation on the dynamic equilibrium.

Upon numerical solution of the truncated system of ordinary differential equations in terms of $q_{2n-1}(t)$, functions $q_{2n}(t)$ should be determined, so that the potential φ associated with sloshing is completely defined. To calculate functions $q_{2n}(t)$, it is straightforward to use Eqs. (15) and (16), which express $q_{2n}(t)$ in terms of $\ddot{q}_{2n-1}(t)$, $\dot{q}_{2n-1}(t)$ and the acceleration of the external excitation $\ddot{X}(t)$. Implications may arise when the second derivatives of $q_{2n}(t)$ are calculated, to compute hydrodynamic pressure and forces. This requires calculation of the fourth derivatives of $q_{2n-1}(t)$ and $X(t)$. In the case of seismic input, $\ddot{q}_{2n-1}(t)$ and $\ddot{X}(t)$ are irregular functions, in the form of a ground acceleration seismic record, containing very sharp variations within very small time intervals and their numerical differentiation may lead to erroneous results. It is possible to avoid such a numerical difficulty, under the observation that vector $\{\gamma\}$ consists of the same elements with the first column of matrix $[\mathbf{M}]$. Therefore, Eq. (20) can be written as follows:

$$[\mathbf{M}]\{\ddot{Q}\}+[\mathbf{K}]\{q\}=0 \tag{25}$$

where

$$\{\ddot{Q}\}=\begin{bmatrix}\ddot{q}_1-\ddot{X}\\ \ddot{q}_3\\ \ddot{q}_5\\ \ddot{q}_7\\ \vdots\end{bmatrix} \tag{26}$$

On the other hand, conditions (15) and (16) can be written

$$\{\overline{q}\}=[\mathbf{D}]\{\ddot{Q}\} \tag{27}$$

where $\{\overline{q}\}$=vector with components $q_{2n}(t)$, $n=1,2,3,\ldots$; and $[\mathbf{D}]$=diagonal matrix with elements

$$D_{kk}=\frac{1}{(2k+1)g}, \quad k=1,2,3,\ldots \tag{28}$$

Combining Eqs. (25) and (27), vector $\{\overline{q}\}$ is calculated as follows:

$$\{\overline{q}\}=-[\mathbf{D}][\mathbf{M}]^{-1}[\mathbf{K}]\{q\} \tag{29}$$

so that functions $q_{2n}(t)$ are directly expressed in terms of functions $q_{2n-1}(t)$. Thus, the double differentiation of functions $q_{2n}(t)$ becomes a trivial procedure.

To account for dissipation effects, a damping term proportional to the first derivative of the generalized coordinates is introduced artificially in Eq. (20), so that the system of ordinary differential equations becomes

$$[\mathbf{M}]\{\ddot{q}\}+[\mathbf{C}]\{\dot{q}\}+[\mathbf{K}]\{q\}=\{\gamma\}\ddot{X} \tag{30}$$

where $[\mathbf{C}]$=square matrix. Herein, matrix $[\mathbf{C}]$ is chosen in the form of Rayleigh damping matrix

$$[\mathbf{C}]=\upsilon[\mathbf{M}] \tag{31}$$

where υ=proportionality constant to be further discussed in section 2.4.

It is also noted that in previous works (Faltinsen 1978; Isaacson and Subbiach 1991), a different formulation was proposed to study damped systems, which enables the use of potential theory and introduces a damping term in the dynamic boundary condition. If that approach is applied in the present problem under consideration, it yields an (ODE) system identical to that of Eq. (30). However, the approach violates basic principles of potential flow, and due to the lack of rigorous theoretical proof, it is not adopted in the present study.

Hydrodynamic Pressures and Forces

Once the velocity potential φ associated with sloshing is calculated, the hydrodynamic pressure at any location can be computed from the linearized Bernoulli equation

$$P(r,\theta,\Psi,t)=-\rho\frac{\partial\Phi}{\partial t}=-\rho\frac{\partial f}{\partial t}-\rho\frac{\partial\varphi}{\partial t} \tag{32}$$

On the right-hand side of Eq. (32) the first term is due to the uniform motion potential, while the second term refers to sloshing effects. The total horizontal force acting on the container is obtained by an appropriate integration of the pressure on the hemispherical wall as

$$F=\int_A P(R,\theta,\Psi,t)(\mathbf{e}_x\cdot\mathbf{n})\,dA \tag{33}$$

where A=hemispherical "wet" surface; $\mathbf{e}_x$=unit vector in the x direction; and $\mathbf{n}$=outer unit vector normal to A. The total force can be also expressed as a summation of the "uniform motion" force

$$F_U=-\rho\int_A\frac{\partial f}{\partial t}(\mathbf{e}_x\cdot\mathbf{n})\,dA \tag{34}$$

or, using Eq. (8)

$$\begin{aligned}F_U&=-\rho R^3\ddot{X}(t)\int_0^{2\pi}\int_0^{\pi/2}\sin^3\theta\cos^2\Psi\,d\theta\,d\Psi\\&=-\left(\frac{2}{3}\pi\rho R^3\right)\ddot{X}(t)=-M_L\ddot{X}\end{aligned} \tag{35}$$

where M_L=total liquid mass of the half-full container, and the force associated with sloshing is

$$F_S=-\rho\int_A\frac{\partial\varphi}{\partial t}(\mathbf{e}_x\cdot\mathbf{n})\,dA \tag{36}$$

or, using Eq. (14)

$$F_S=-\rho R^3\pi\sum_{n=1}^{\infty}R^{2n-2}\{\ddot{q}_{2n-1}(t)Y_{2n-1}+R\ddot{q}_{2n}(t)Y_{2n}\} \tag{37}$$

where

$$Y_s=\int_0^{\pi/2}P_s^1(\cos\theta)\sin^2\theta\,d\theta,\quad s=1,2,3,\ldots \tag{38}$$

Since the pressure is always normal to the wall of the container, the total hydrodynamic force direction always passes through the center of the sphere.

Note that the only numerical work required to obtain the solution is related to the solution of a truncated system of ordinary linear differential equations. As shown in the section "Numerical Results and Discussion," a relatively small truncation size N ($n\leqslant N$) is adequate to obtain good results.

Simplified Formulation

It is possible to develop a simplified version of the above formulation considering only the first term ($N=1$) of the series expansion of the potential φ associated with sloshing in Eq. (14). In this case, φ is assumed equal to

$$\varphi(r,\theta,\Psi,t)=[\dot{q}_1rP_1^1(\mu)+\dot{q}_2r^2P_2^1(\mu)]\cos\Psi \tag{39}$$

Applying the boundary conditions on the free surface and the container wall, the final equation, analogous to Eq. (20), is equal to

$$\ddot{q}_1+\left(\frac{4g}{3R}\right)q_1=\ddot{X} \tag{40}$$

The other unknown function q_2 is defined by the following equation, analogous to Eq. (15):

$$q_2=\frac{1}{3g}[\ddot{q}_1-\ddot{X}] \tag{41}$$

which results in

$$q_2=-\left(\frac{4}{9R}\right)q_1 \tag{42}$$

an expression analogous to Eq. (29).

To account for dissipation effects, a damping term is introduced

$$\ddot{q}_1+v\dot{q}_1+\left(\frac{4g}{3R}\right)q_1=\ddot{X} \tag{43}$$

which is an equation analogous to Eq. (30). It is possible to rewrite Eq. (40) in the form of a linear oscillator with viscous damping

$$\ddot{q}_1+2\xi_S\omega_S\dot{q}_1+\omega_S^2q_1=\ddot{X} \tag{44}$$

where

$$\omega_S=\sqrt{4g/3R} \tag{45}$$

is the circular (undamped) frequency and

$$\xi_S=v/2\omega_S \tag{46}$$

is the damping ratio. Regarding the ω_S value, it is an approximation of the first sloshing frequency ω_1, since only one term of the series expansion is employed ($N=1$). The dependence of sloshing frequency values on the truncation size N will be discussed in detail in the section "Numerical Results and Discussion." Moreover, Eq. (46) can be employed to determine the value of v, if the damping ratio ξ_S is somehow measured (e.g., experimentally).

The hydrodynamic pressure on the wall is calculated from Eq. (32) and the corresponding force becomes

$$F=F_U+F_S=-M_L\ddot{X}+\frac{M_L}{2}\ddot{q}_1=-M_L\ddot{X}+M_S\ddot{q}_1 \tag{47}$$

In the above equation, M_S expresses the part of liquid mass associated with sloshing motion. The form of Eqs. (44) and (47) motivates the development of a simple mechanical model to approximate the response of a half-full spherical container. Setting

$$\begin{aligned}u_1&=(-q_1)+X\\u_2&=X\end{aligned} \tag{48}$$

it is straightforward to rewrite Eqs. (44) and (47) in the following manner:

$$M_S\ddot{u}_1+\upsilon M_S(\dot{u}_1-\dot{u}_2)+\omega_S^2 M_S(u_1-u_2)=0 \tag{49}$$

$$F=-M_S\ddot{u}_1-(M_L-M_S)\ddot{u}_2=-\frac{M_L}{2}\ddot{u}_1-\frac{M_L}{2}\ddot{u}_2 \tag{50}$$

Based on Eqs. (49) and (50), the proposed mechanical model illustrates the motion of the fluid-container system. In this model, u_2 represents the motion of the external source, and u_1 expresses the motion of the liquid mass associated with sloshing. In addition, the total liquid mass M_L is split in two equal parts m_1 and m_2, which correspond to u_1 and u_2, and express the so-called ''convective'' (or ''sloshing'') mass and ''impulsive'' motion, respectively, a concept introduced by Housner (1957). It should be noted that based on the proposed mathematical formulation, more elaborate mechanical models can be proposed, which include more than one oscillator. Those models permit a more accurate description of sloshing, but their development is not within the scope of the present work.

Solution for Harmonic Excitation

The mathematical formulation for arbitrary excitation is significantly simplified when the rigid container undergoes a harmonic motion

$$\dot{X}(t)=Ue^{-i\omega t} \tag{51}$$

where U=velocity amplitude; and ω=angular frequency of the external excitation source. Assuming steady-state conditions, the total velocity potential $\Phi(x,y,z,t)$ is written as

$$\Phi(x,y,z,t)=[f(x)+\varphi(x,y,z)]e^{-i\omega t} \tag{52}$$

where

$$f(x)=Ux=Ur\sin\theta\cos\Psi \tag{53}$$

and

$$\varphi(x,y,z)=\sum_{n=1}^{\infty}[a_{2n-1}P_{2n-1}^1(\mu)r^{2n-1}+a_{2n}P_{2n}^1(\mu)r^{2n}]\cos\Psi \tag{54}$$

are the uniform motion potential and the potential related to sloshing, respectively. Eq. (54) can be readily obtained from Eq. (14) assuming harmonic functions for $\dot{q}_n(t)$

$$\dot{q}_n(t)=a_n e^{-i\omega t} \tag{55}$$

More specifically, applying the corresponding boundary conditions, the following relations are obtained [compare with Eqs. (15) and (16)]:

$$a_2=-\frac{1}{3}\frac{\omega^2}{g}a_1+\frac{1}{3}\frac{\omega^2}{g}U \tag{56}$$

and

$$a_{2n}=-\frac{1}{(2n+1)}\frac{\omega^2}{g}a_{2n-1}, \quad n>1 \tag{57}$$

Consequently, the following infinite system of linear algebraic equations is obtained:

$$(-\omega^2[\mathbf{M}]+[\mathbf{K}])\{a\}=-\omega^2U\{\gamma\} \tag{58}$$

In the above system, the square matrix $[\mathbf{M}]$, the diagonal matrix $[\mathbf{K}]$, and the vector $\{\gamma\}$ are given by Eqs. (22), (23), and (24), respectively, and $\{a\}$ is the unknown vector with components a_{2n-1}, $n=1,2,3,\ldots$. Once the system is solved and the unknown coefficients are computed, the velocity potential φ is determined from Eq. (54) using Eqs. (56) and (57).

Similarly, the response of the damped system is obtained through the solution of the following algebraic system:

$$(-\omega^2[\mathbf{M}]-i\upsilon\omega[\mathbf{M}]+[\mathbf{K}])\{a\}=-\omega^2U\{\gamma\} \tag{59}$$

Upon calculation of a_n, $n=1,2,3,\ldots$, the hydrodynamic pressures and the force acting on the container can be computed from Eqs. (32) and (33), respectively. It is interesting to note that in the case of harmonic excitation the only computational work required is the solution of a truncated linear algebraic system. Note that when $U=0$, the system of algebraic Eq. (58) is reduced to a homogeneous system, which is identical to the one obtained by Evans and Linton (1993).

An estimate of the externally induced sloshing effects on the overall response can be obtained from the computation of the added mass coefficient, defined as follows:

$$C_a=\mathrm{Re}\left[\frac{F_S}{F_U}\right] \tag{60}$$

On the other hand, the dimensionless damping coefficient

$$C_v=\mathrm{Im}\left[\frac{F_S}{F_U}\right] \tag{61}$$

provides a measure of the dissipation mechanism when damping is included (Isaacson and Subbiach 1991). In the above expressions, Re[ ] and Im[ ] denote the real and the imaginary part of the F_S/F_U ratio, respectively.

It is possible to obtain an elegant analytical solution for harmonic excitation, if the sloshing potential $\varphi(x,y,z)$ is approximated only with the first term ($N=1$) of the series expansion. The system of Eq. (58) now reduces to a scalar equation in terms of the unknown coefficient a_1. The remaining unknown coefficient a_2 is computed from Eq. (56). The resulting expressions are substituted into Eq. (54) to obtain the following closed-form expression for the velocity potential φ:

$$\varphi(r,\theta,\Psi,t)=\frac{U\omega^2}{\left(\frac{4g}{3R}-\omega^2\right)-i\upsilon\omega}\left(1-\frac{4r}{3R}\cos\theta\right)\times r\sin\theta\cos\Psi e^{-i\omega t} \tag{62}$$

Furthermore, the force F_S corresponding to sloshing becomes

$$F_S=i\omega U\frac{1}{3}\pi R^3\rho\frac{\omega^2}{\left(\frac{4g}{3R}-\omega^2\right)-i\upsilon\omega}e^{-i\omega t} \tag{63}$$

Finally, the added mass coefficient and the dimensionless damping coefficient are

$$C_a=\frac{1}{2}\frac{\omega^2\left(\frac{4g}{3R}-\omega^2\right)}{\left(\frac{4g}{3R}-\omega^2\right)^2+(\upsilon\omega)^2}=\frac{1}{2}\frac{\lambda^2(1-\lambda^2)}{(1-\lambda^2)^2+(2\lambda\xi_S)^2} \tag{64}$$

and

$$C_v=\frac{1}{2}\frac{\upsilon\omega^3}{\left(\frac{4g}{3R}-\omega^2\right)^2+(\upsilon\omega)^2}=\frac{1}{2}\frac{2\lambda^3\xi_S}{(1-\lambda^2)^2+(2\lambda\xi_S)^2} \tag{65}$$

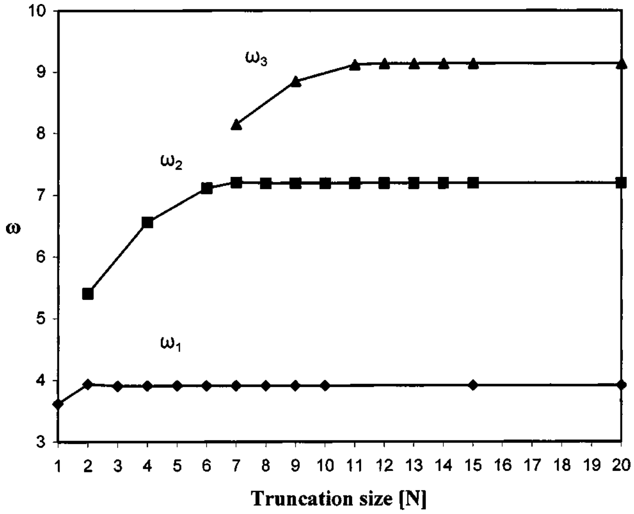

Fig. 2. Variation of first three eigenfrequencies with respect to truncation size N for $\upsilon=0$ ($R=1$, $g=9.81$)

respectively, where $\lambda=\omega/\omega_S$=ratio of the external frequency over the natural frequency of the oscillator.

Numerical Results and Discussion

The numerical results presented in this section are based on the solution of Eqs. (58) and (20) for the cases of harmonic and arbitrary excitation respectively. The convergence of the solution is always tested, increasing the truncation size of the expansion, so that accurate results up to certain significant figures are obtained. In all cases analyzed, despite the fact that the series solution is expressed in terms of nonorthogonal functions, the convergence of the solution is rapid.

Sloshing Frequencies

To establish some confidence in the present formulation the eigenvalue problem is considered first, assuming no external excitation $[X(t)=0]$. The convergence rate and the expected accuracy of the eigenvalue solution are demonstrated numerically, increasing the value of truncation size N. In Fig. 2, the variation of the first three eigenvalues ω_1, ω_2, and ω_3, in terms of the truncation size N for zero dissipation $(\upsilon=0)$ is shown. The results indicate that the convergence rate is quite rapid. Furthermore, faster convergence is obtained in lower sloshing frequencies. The required truncation size N to obtain accurate results up to three significant figures for ω_1, ω_2, and ω_3 is $N=3$, $N=8$, and $N=12$, respectively. It is interesting to note that the ω_S value is 3.617 [derived analytically Eq. (45)], offers a reasonable estimate of the converged value of the first eigenfrequency ω_1 (3.912). It is noted that the converged ω_1, ω_2, and ω_3 values from the present analysis are identical to those reported by McIver (1989) and Evans and Linton (1993). Furthermore, the converged values of ω_1, ω_2, and ω_3 are in very good agreement with the experimental results of Abramson et al. (1963), and with unpublished data from tests conducted by Lockheed Missile Systems Division, as reported by Budiansky (1960).

When damping is present $(\upsilon\neq 0)$ the eigenvalues of the system become complex because of energy dissipation effects. The convergence of the complex eigenfrequencies ω_1, ω_2, and ω_3 of the damped system is similar to the corresponding eigenfrequencies of the undamped system. Some typical results for $\upsilon=0.72$ are shown in Fig. 3, where the convergence of the real and imaginary parts of the first and the second sloshing frequency is demonstrated.

It should be underlined that the sloshing frequency values in hemispherical containers depend on the truncation size of the series expansion N, due to the nonorthogonality of the spatial functions $\varphi_n(x,y,z)$ in Eqs. (13) and (14). On the other hand, when a similar series solution approach [analogous to Eq. (13)] is applied in vertical cylinders or rectangles, mutually orthogonal functions $\varphi_n(x,y,z)$ are employed, so that the corresponding sloshing modes are uncoupled (Currie 1974).

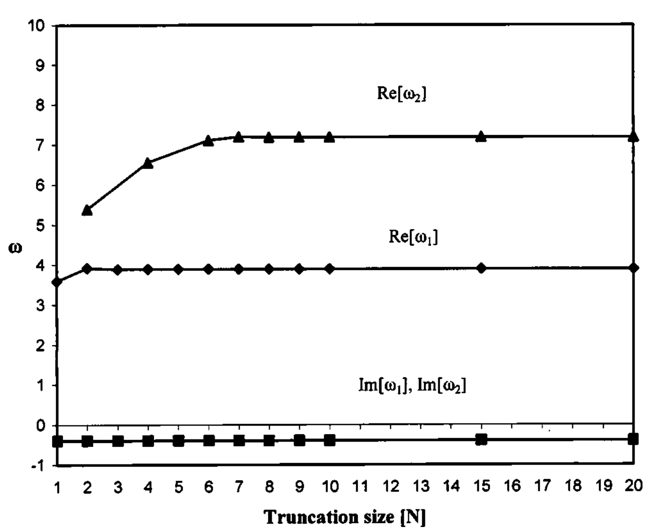

Fig. 3. Variation of first and second eigenfrequency with respect to truncation size N for $\upsilon=0.72$ ($R=1$, $g=9.81$)

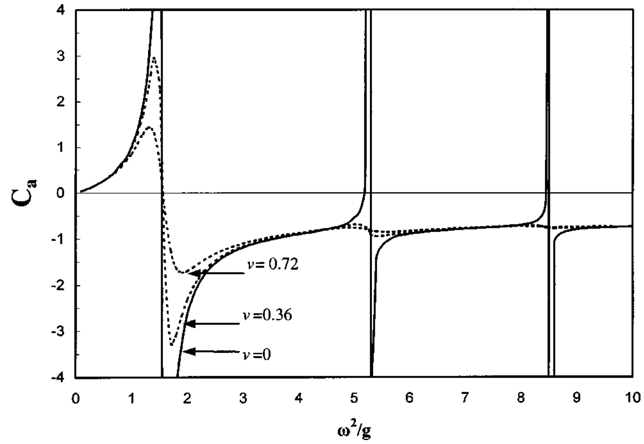

Fig. 4. Converged value of C_a in terms of external excitation frequency (ω^2/g) for $N=50$ ($R=1$)

Hydrodynamic Forces under Harmonic Excitation

The case of harmonic excitation of a half-full sphere is investigated next. The corresponding results are shown in terms of the added mass coefficient C_a and the dimensionless damping coefficient C_υ. These coefficients can be used for assessing the effects of sloshing for a wide range of external source frequencies.

Table 1. Convergence of C_a versus N ($\upsilon=0$, $R=1$); Converged Value of ω_1^2/g is 1.5601

ω^2/g N	1	1.3	1.6	1.9
1	1.5000	19.500	−3.0000	−1.6764
2	1.0516	2.8712	−53.400	−3.9075
3	1.0476	2.9580	−20.543	−3.1687
4	1.0430	2.9268	−22.199	−3.2180
5	1.0410	2.9160	−22.644	−3.2228
10	1.0393	2.9044	−23.176	−3.2299
15	1.0391	2.9031	−23.241	−3.2307
20	1.0390	2.9027	−23.254	−3.2309

Table 2. Convergence of C_a versus N ($\upsilon=0$, $R=1$); Converged Value of ω_2^2/g is 5.2753

ω^2/g N	4.8	5.1	5.4	5.7
1	−0.6923	−0.6769	−0.6639	−0.6526
2	−0.7440	−0.7350	−0.7258	−0.7071
3	−0.7588	−0.7467	−0.7378	−0.7304
4	−0.7950	−0.7816	−0.7668	−0.7537
5	−0.7357	−0.6948	−0.6926	−0.7087
10	−0.7049	−0.4008	−1.4348	−0.9841
15	−0.7050	−0.4030	−1.4375	−0.9840
20	−0.7050	−0.4037	−1.4383	−0.9839

Table 3. Convergence of C_a versus N ($\upsilon=0$, $R=1$); Converged Value of ω_3^2/g is 8.5040

ω^2/g N	8.1	8.4	8.7	9.0
1	−0.5985	−0.5943	−0.5904	−0.5869
2	−0.6665	−0.6621	−0.6581	−0.6544
3	−0.6881	−0.6841	−0.6803	−0.6768
4	−0.6986	−0.6946	−0.6908	−0.6873
5	−0.7049	−0.7010	−0.6972	−0.6937
10	−0.6871	−0.6255	−0.5747	−0.6247
15	−0.6800	−0.4433	−0.9023	−0.7970
20	−0.6801	−0.4450	−0.9023	−0.7969

The convergence of the C_a value is demonstrated for a unit-radius sphere ($R=1$) and zero damping ($\upsilon=0$) in Tables 1, 2, and 3, for different values of the truncation size N and for different values of external excitation frequencies (expressed in terms of ω^2/g). Each table refers to a range that includes one of the first three natural frequencies of the system, so that the convergence of C_a is examined in the vicinity of the natural frequencies. In the majority of the cases analyzed, the results converge up to three significant figures for $N\leqslant 10$. As expected, a larger truncation size N is required when the external frequency approaches each of the resonant frequencies of the system.

Subsequently, the added mass coefficient C_a and the dimensionless damping coefficient C_υ are plotted as functions of the external excitation frequency (ω^2/g) in Figs. 4 and 5, respectively, for three different values of the damping parameter υ equal to 0, 0.36, and 0.72 ($R=1$, $g=9.81$). According to Eq. (46), the three values of υ correspond to 0, 5, and 10% values of damping ratio ξ_S, respectively. Fig. 4 shows that for the case of zero damping, the response is characterized by large increases in the added mass coefficient C_a in the vicinity of resonant frequencies.

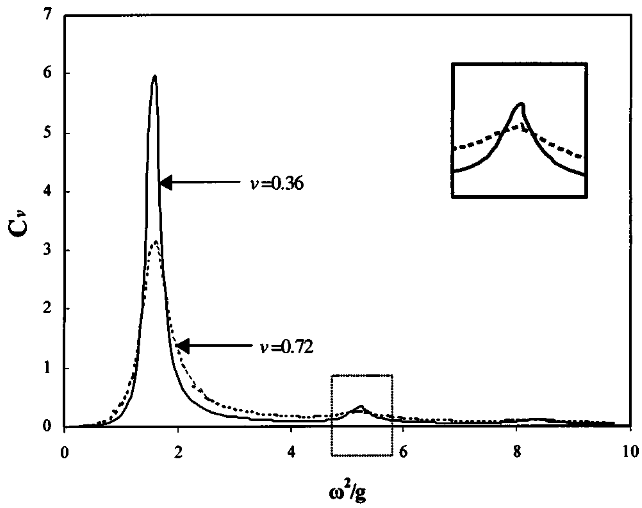

Fig. 5. Converged value of C_υ in terms of external excitation frequency (ω^2/g) for $N=50$ ($R=1$)

There is a sign reversal in C_a at each resonant frequency. When $C_a\leqslant 0$ the sloshing force F_S opposes the "uniform motion" force F_U resulting in a reduction of the total force amplitude. The extreme values of the added mass coefficient close to the resonant frequencies are significantly reduced when damping is present, and the resonant effect of the higher natural frequencies almost disappears. The large values of C_a for a wide range of excitation frequencies indicate the significant effects of hydrodynamic sloshing on the overall response. Fig. 5 presents the corresponding results for the dimensionless damping coefficient C_υ. The C_υ value exhibits a peak near the first resonant frequency, and much smaller peaks for the higher resonant frequencies. When the damping parameter value is increased, the peaks become smaller and smoother.

Results for Earthquake Ground Motion

The response of hemispherical liquid containers under earthquake excitation is of particular importance for the seismic analysis of spherical pressure vessels used in refineries and petrochemical

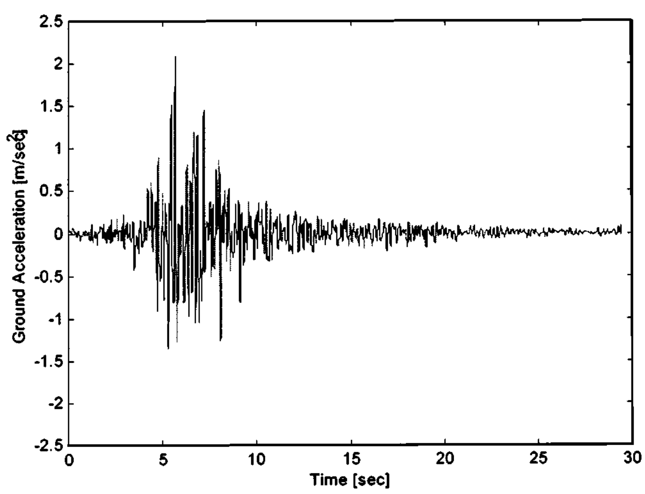

Fig. 6. Ground acceleration record for Kozani earthquake, Northern Greece, 1995

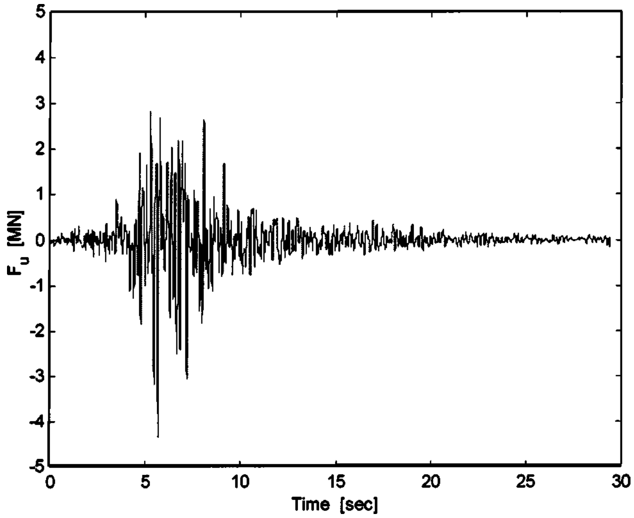

Fig. 7. Force associated with uniform motion ($R=10$ m, $g=9.81$ m/s^2, $\rho=1{,}000$ kg/m^3)

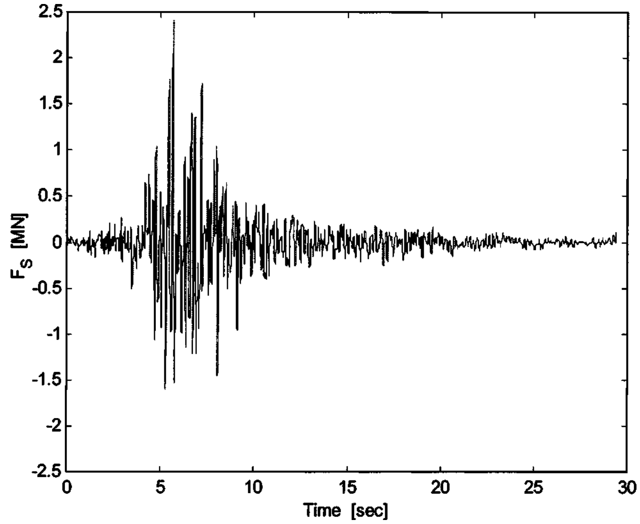

Fig. 8. Force associated with sloshing for $\upsilon=0$ (undamped system) and truncation size $N=2$ ($R=10$ m, $g=9.81$ m/s^2, $\rho=1{,}000$ kg/m^3)

industries. In the present paper, the seismic ground motion occurred in Kozani, Greece, in 1995, is considered (Fig. 6). The linear system of ordinary differential equations is integrated in time by implementing a fourth-order Runge-Kutta scheme in *Matlab* programming. Following a short parametric study, the time step Δt is chosen equal to 0.005 s.

The dependence of the value of the maximum total force $F_{\max}$ on the truncation size N is indicated in Table 4. The results indicate that consideration of sloshing in the analysis has a very significant effect on the maximum value of the total force $F_{\max}$. It is also shown that consideration of few terms of the series solution (e.g., $N=2$) is adequate to provide quite accurate results in terms of the $F_{\max}$ value, for engineering purposes.

Figs. 7, 8, and 9 show the variation of the uniform force F_U, the force associated with sloshing F_S, and the total force F, respectively, for a half-full sphere subjected to the Kozani earthquake and for zero damping ($\upsilon=0$). The sphere has a radius R equal to 10 m, and contains a liquid of density ρ equal to 1,000 kgr/m^3 ($g=9.81$ m/s^2). A truncation size N equal to 2 is considered in this analysis. The results show that the maximum value of the uniform motion force $F_{U\max}$ is significantly larger than the maximum value of the total force $F_{\max}$ and that sloshing effects result in a reduction of the total hydrodynamic force. The fact that the dominant earthquake frequencies are significantly larger than the sloshing frequencies offers a reasonable explanation for the beneficial effect of sloshing. More specifically, under these conditions the sloshing force F_S opposes the uniform motion force F_U and, therefore, the total force is reduced.

Figs. 10 and 11 show the half-full vessel response in terms of forces under the Kozani earthquake, for 10% damping (the value of υ is chosen equal to 0.228, so that $\xi_S=10\%$). Due to the high values of dominant earthquake frequencies as opposed to the low values of the sloshing frequencies, the sloshing force $F_{S\max}$ is found equal to 2.402 MN. A comparison with the corresponding maximum sloshing force obtained from the analysis of the undamped system (2.408 MN) indicates that the maximum sloshing force is almost unaffected by the presence of damping.

A more detailed presentation of damping effects is demonstrated in Figs. 12 and 13, where the time history of the generalized coordinates $q_1(t)$ and $q_3(t)$ for zero damping ($\upsilon=0$) and for 10% damping ($\upsilon=0.228$) is shown. The presence of damping results in a significant attenuation of the $q_1(t)$ and $q_3(t)$ values.

Table 4. Dependence of Maximum Total Force Value ($F_{\max}$) in Terms of Truncation Size N (Undamped System)

N	F (max) (MN)
0	4.365
1	2.201
2	1.958
3	1.874
4	1.870

Conclusions

A mathematical model is developed for externally induced liquid sloshing in half-full spherical containers. The velocity potential is split in two parts, a "uniform motion" potential (trivially ob-

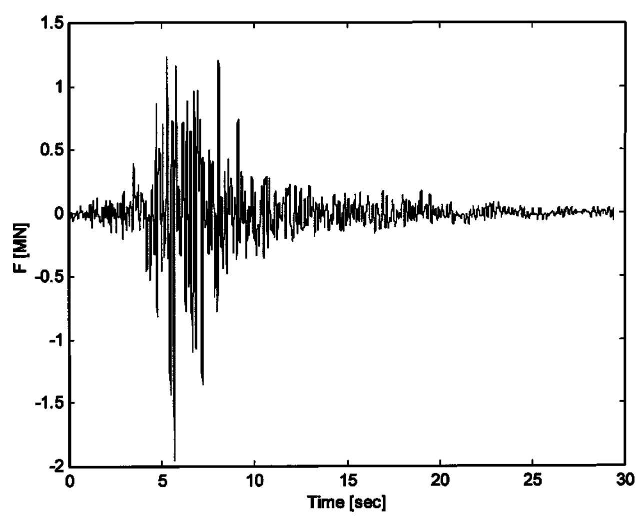

Fig. 9. Total force for $\upsilon=0$ (undamped system) and truncation size $N=2$ ($R=10$ m, $g=9.81$ m/s^2, $\rho=1{,}000$ kg/m^3)

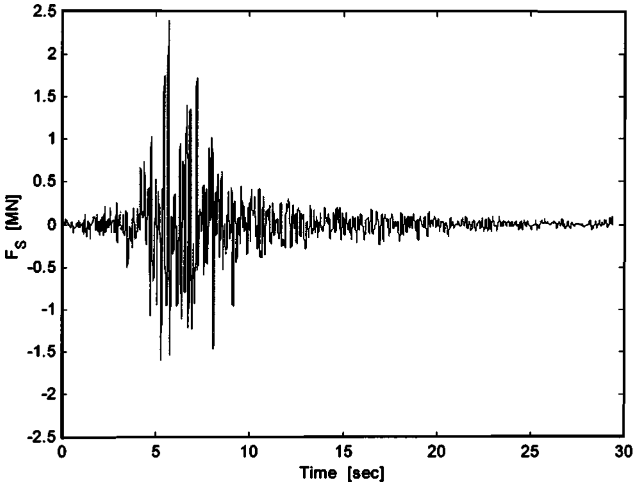

Fig. 10. Force associated with sloshing for $v=0.228$ ($\xi_s=10\%$) and truncation size $N=2$ ($R=10$ m, $g=9.81$ m/s^2, $\rho=1{,}000$ kg/m^3)

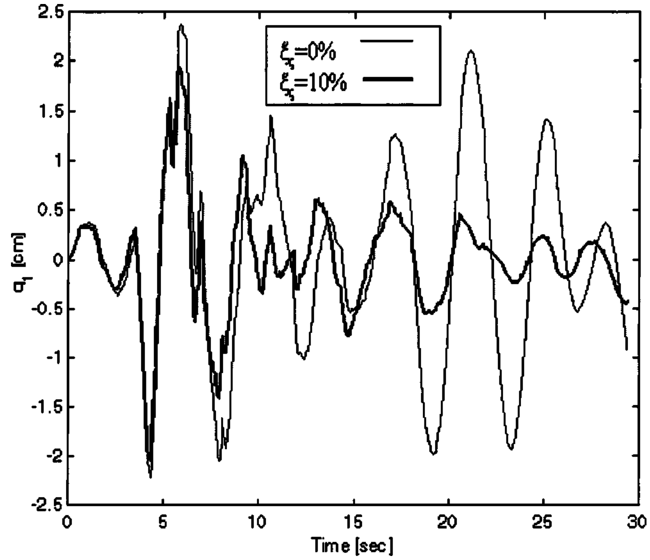

Fig. 12. Time history of generalized coordinate q_1 for 0 and 10% damping ($R=10$ m, $g=9.81$, m/s^2 $\rho=1{,}000$ kg/m^3)

tained) and a potential associated with sloshing. In this configuration, the problem formulation is not separable and the general solution of the sloshing potential is written as a series expansion of arbitrary time functions and its associated nonorthogonal spatial functions. Furthermore, viscous damping effects can be taken into consideration. The formulation reduces in a system of linear differential equations, which is solved numerically.

The present formulation enables the prediction of sloshing effects in hemispherical liquid containers under any form of external excitation, in a simple and efficient manner. The problem is significantly simplified if only the first term of the series is considered, and a mechanical model is proposed to describe sloshing response. For the particular case of a harmonic external source, closed-form expressions for the sloshing potential and the sloshing force are obtained.

A numerical investigation of convergence is conducted to determine the sensitivity of results on the truncation size of the series solution. It is found that higher sloshing frequencies require a larger truncation size to achieve convergence. In the case of harmonic excitation, the results are expressed in terms of the added force coefficient and the dimensionless damping coefficient, and indicate that convergence is less rapid in the vicinity of resonant frequencies, and that the presence of damping diminishes resonant effects.

Subsequently, the response of a spherical vessel subjected to a real seismic event is examined, and the total horizontal force acting on the container is calculated. The results indicate that sloshing has a significant effect on the value of the total force. On the other hand, the numerical results demonstrate that few sloshing terms in the series solution are adequate so that very good results are obtained. It is also found that consideration of sloshing results in a reduction of the total force with respect to the "uniform motion" force, because the dominant earthquake frequencies are significantly higher than the sloshing frequencies. For the same

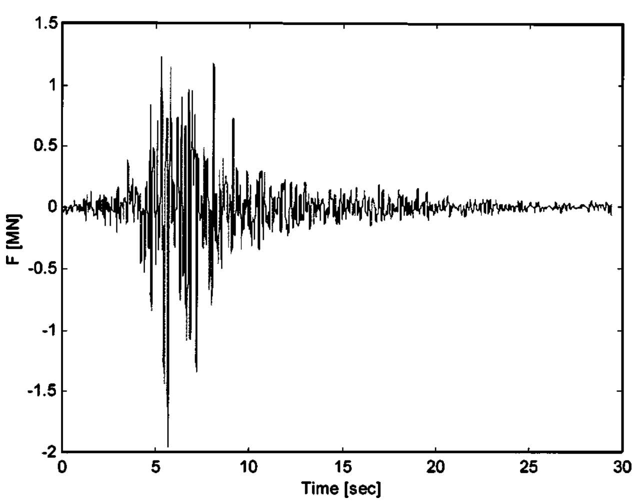

Fig. 11. Total force for $v=0.228$ ($\xi_s=10\%$) and truncation size $N=2$ ($R=10$ m, $g=9.81$ m/s^2, $\rho=1{,}000$ kg/m^3)

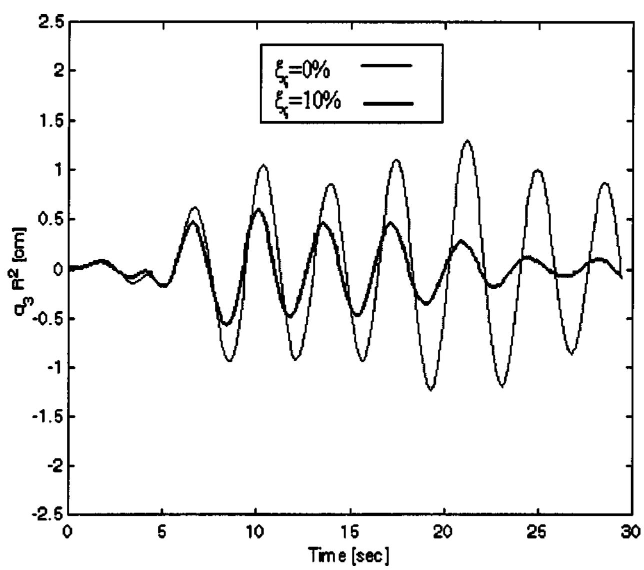

Fig. 13. Time history of generalized coordinate q_3 for 0 and 10% damping ($R=10$ m, $g=9.81$ m/s^2, $\rho=1{,}000$ kg/m^3)

reason, the effects of viscous damping have an insignificant effect on the maximum value of the sloshing force.

Acknowledgments

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